Uncertainty Based Methods for Analysis and Design of Aerospace Systems

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Outline

- Motivation for UBM
- Capabilities and Framework
- Methods and Tools
- Example Problems
- Applications to IRAC
Motivation

Increase confidence and consistency in vehicle stability and performance predictions by developing improved methods for quantifying and managing uncertainty

Quantifying

• Uncertainty Modeling
  – Given data from an uncertain system how best to model observed variability?

• Uncertainty Propagation
  – Given models of variability how do they propagate through system models?

Managing

• Generate designs that robustly accommodate for uncertainty
Expected Results/Goals

- Safer more reliable and robust systems designed without unnecessary conservatism
- Account for the likelihood of all possible outcomes
• Discipline independent $\rightarrow y = M(d,p)$
  $\Rightarrow M$ could be algebraic, ODE, PDE, or a multi-disciplinary code.

• Uncertainty Types:
  $\Rightarrow$ Parameter uncertainty: $p$
  $\Rightarrow$ Model uncertainty: $M$
Uncertainty Propagation

System Model, $\mathbf{M}$, maps parameters, $p$, to performance metrics, $y$.

Intervals map to intervals

Density functions map to density functions
Common Pathologies

• Parameter extremes not necessarily performance extremes,

\[ y^+ \succcurlyeq M(p^+, d) \text{ and } y^- \preccurlyeq M(p^-, d) \]

• also

\[ E[y] = E[M(p, d)] \succcurlyeq M(E[p], d) \]

• Bounded parameters can lead to unbounded performance
Probabilistic Measures of Performance

• PDF of performance usually not explicitly computed, instead statistical moments and failure probabilities are computed

  – **Mean** = average value of system performance, $E[y]$
  – **Variance** = measure of “spread” of performance, $V[y]$
  – **Probability of failure** = probability of exceeding a prescribed range, $P[y<y^*]$
Probabilistic Design

Given $M$ and $f_P(p)$ we want to determine $d$ such that $f_y(y)$ is shaped as desired.

- **Design Goals**
  - Target-based
  - Reliability-based

Smallest variability about a target value

Minimal or no violation of inequality constraints
Probabilistic Control Design

Uncertain parameters

Design variables $p_1, p_2$ $\rightarrow$ Closed-loop system

Random process in time

Control Saturation

Random process in frequency

Instability

Complex random variables

High frequency

Low frequency

Random process in time

Overshoot

Settling time

Probabilistic Control Design
Methods and Tools
Existing Capabilities

MATLAB-based environment with the following capabilities:

• **Probabilistic analysis tools**
  – First- and Second-Order Reliability Methods (FORM/SORM)
  – Deterministic sampling and hybrid methods
  – Efficient moment propagation methods

• **Probabilistic design tools**
  – Reliability formulations with shapeable failure domains
  – Efficient probabilistic design tools based on bounds to failure probabilities
  – Moment-based strategies for robust design

• **Support tools**
  – Response surface generator with several basis functions
  – Probabilistic decomposition for optimal risk mitigation
Deterministic Sampling VS Monte Carlo

- Monte Carlo sampling
  - Pseudo-random samples

- Deterministic sampling
  - Substantial gains in efficiency and accuracy
  - Fixed pattern samples, distributed uniformly over the sample space
    - Hammersley Sequence Sampling
    - Halton Sequence (Leaped)
- Can be used to approximate failure probability
Advantages of Deterministic Sampling

Example: Find mean of a cubic function of 3 uniform [0,1] RVs.

- Better accuracy for a fixed number of samples
- Better efficiency for a fixed relative error
Homothetic Deformations

- Monte Carlo sampling
- Homothetic deformations
  - Optimization-based
  - Computationally cheap
  - Analytical expressions for $P[H]$ 

Applications

1. Robustness metric $\rightarrow$ PSM
2. Upper bounds to $P[F]$
3. Hybrid Method for $P[F]$
Hybrid Method: Efficiency & Accuracy

Monte Carlo sampling vs. Hybrid Method

Improvement in efficiency:
\[ \eta_{mc} = \eta \eta_{hm} \]

Improvement in accuracy:
\[ \Delta_{mc} = \eta \Delta_{hm} \]
Applications
Validation Challenge Workshop

Sandia National Laboratory Workshop
(13 international teams of experts invited to participate)

Challenge: Adequate statistical characterization of uncertainty using limited experimental data.
LaRC’s Solution

Statistical matching of parametric uncertainties

Statistical response of uncertainty model versus data

Results on Target System

Frequency, Hz

\begin{align*}
\text{Accel at mass 3, in/sec}^2 \\
\text{Frequency, Hz}
\end{align*}

\begin{align*}
\text{Mean Matching Only} \\
\text{Mean and Confidence Interval Matching}
\end{align*}

\begin{align*}
\text{P}[F] &= 0.20 \\
\text{P}[z > 18000 \text{ in/sec}^2] &< 0.01
\end{align*}

Regulatory Requirement
Flexible Beam

Disturbance rejection for a flexible beam test article
• Physical and modal parameter uncertainty
• Finite element-based model (5 elastic modes)
• SISO system (hub motor torque → tip velocity)
• Filtered white noise is used to model the disturbance.

Uncertainty Sources
\[ p = [E, \rho, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5] \]

Compensator Structure
• Full-state feedback & full-order observer

Design Requirements
• Stability and output RMS
Beam: RMS Closed-loop Performance

- RMS value
- Reliability-based design
- Target-based design
- Probability density function
- Optimal deterministic value
Robust Control Analysis

- **Problem**: Benchmark robust control challenge posed in the ACC
- **Plant**: nonlinear with a non-collocated sensor-actuator pair
- **Uncertain parameters**
  - Masses
  - Linear stiffness
  - Nonlinear stiffness
  - Time-delay
  - Control effectiveness (failure)
- **Closed-loop requirements**
  - Local stability
  - Settling time
  - Control usage
- **Controllers**
  - 11 controllers were designed by different authors using different methods
- **Comparative analysis of these compensators**
Robust Control Analysis

- **Fixed Uncertainty Models**
  - Monte Carlo

- **Uncertainty Dilations**
  - Dilations consistent with physical constraints
  - Structure of dilation are up to the analyst
  - The uncertainty radius $R$, is proportional to the amount of uncertainty

- **Robust Stability**
  - What is the largest $R$ for which closed-loop system is robustly stable

- **Robust Performance**
  - Evaluate the performance degradation caused by uncertainty

- **Complexity**
  - Is the complexity of the compensator justified by its robustness characteristics?
Robust Control Analysis

Compensator A (3+4 gains)

- Robust Stability

- Robust Performance

\[ J = \int (x^T Q x + u^T R u) dt \]

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Compensator B (4+5 gains)

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Uncertainty Radius
Applications to IRAC

• Critical Parameter ID
  – Determine critical combination of uncertain parameters and failures leading to instability and unacceptable time responses, e.g. what is the largest time-delay an adaptive control system can tolerate before becoming unstable

• Evaluate Margins
  – Determine margins for robust stability and satisfactory performance

• Severity of Failures
  – Determine the severity of individual failure modes and the efficacy of system redundancies

• Performance degradation caused by uncertainty